AD-A014 708

TESTING PROPOSED HEAD/NECK MODELS WITH THE DATA FROM THE INITIAL EXPERIMENT

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Prepared for:

Office of Naval Research

3 July 1974

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Applied Re...rch in Statistics - Mathematics - Operations Research

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TECHNICAL NOTE NO. 5

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Prepared under
Office of Naval Research
Contract N00014-74-C-0154, Task NR 105-757

The two previous technical notes (No. 3 and No. 4) described the use of the data from the initial experiment in development of an injury prediction model. The present technical note, which refers to the discussion and notation of the two previous technical notes, outlines the use of the same data to test the reasonableness of proposed head/neck models.

Background and Assumptions

Within each cell (i,j,k,m) of the initial experiment, there will be vector observations

$$\underline{y}_{ijkmn} = \begin{pmatrix} y_{ijkmn1} \\ \vdots \\ y_{ijkmnP} \end{pmatrix}$$

for n = 1,...,N where the variables y_{ijkmnp} denote the set of variables describing dynamic response severity which were defined in Technical Note No. 3. For purposes of testing a proposed head/neck model, it makes no difference whether these variables are the variables h^* or the transformed variables h.

For a head/neck model, let $\hat{\underline{y}}_{ijkmn}$ denote the vector of the predicted values of $\underline{y}_{ijkmn}$. It will be assumed that these predicted values were determined without using the observed values in the initial experiment. Define the vector of differences between observed and predicted values as

$$\underline{d}_{ijkmn} = \begin{pmatrix} d_{ijkmn1} \\ \cdot \\ \cdot \\ \cdot \\ d_{ijkmnP} \end{pmatrix} = \underline{y}_{ijkmn} - \hat{\underline{y}}_{ijkmn}$$

Assume that $\underline{y}_{ijkmn}$ is from a multivariate Normal distribution with mean vector $\underline{\mu}_{ijkmn}$ and covariance matrix $\underline{V}_1$, and that $\hat{\underline{y}}_{ijkmn}$ is from a multivariate Normal distribution with mean vector $\underline{\gamma}_{ijkmn}$ and covariance matrix $\underline{V}_2$. If model predictions were unbiased, then $\mu_{ijkmn}=\gamma_{ijkmn}$ so that each $\underline{d}_{ijkmn}$ would have an expected value of zero. Thus, a test of the hypothesis $E(\underline{d}_{ijkmn}) = \underline{0}$ will provide information about the usefulness of the model.

Because $\underline{y}_{ijkmn}$ and $\hat{\underline{y}}_{ijkmn}$ are assumed to be multivariate Normal, it follows that the vectors $\underline{d}_{ijkmn}$ constitute a sample of size IJKMN from a multivariate Normal distribution. It should be noted that because observations have been obtained from each subject under all IJK run profiles, there will be correlation between those observations. Fortunately, however, when working with the $\underline{d}$'s, this poses no problem. To illustrate this, consider two observations: y_1 for a particular subject for a given run profile, and y_2 for the same subject for a different run profile. Let $\hat{y}_1$ and $\hat{y}_2$ be the corresponding model predictions. Denote the variance of h_i by σ_i^2 and the correlation between h_1 and h_2 by ρ. If the proposed head/neck model provides valid predictions, it is reasonable to assume that the variance of

$\hat{h}_i$ is approximately σ_i^2 and the correlation between $\hat{h}_1$ and $\hat{h}_2$, between $\hat{h}_1$ and h_2, and between h_1 and $\hat{h}_2$ is approximately ρ. From this it follows that the correlation between $d_1 = h_1 - \hat{h}_1$ and $d_2 = h_2 - \hat{h}_2$ is approximately zero, so that the vectors $\underline{d}_{ijkmn}$ may be regarded as independent. Therefore, if $E(\underline{d}_{ijkmn}) = \underline{0}$, these vectors comprise a random sample of IJKMN independent observations from the same multivariate Normal distribution having zero mean vector.

Statistical Test

In view of the above discussion, Hotelling's T^2 statistic may be used to test the hypothesis that $E(\underline{d}_{ijkmn}) = \underline{0}$. This statistic is given by

$$T^2 = IJKMN(\underline{d}_{.....})'\underline{\hat{V}}^{-1}(\underline{d}_{.....})$$

where $\underline{d}_{.....}$ is the $(P \times 1)$ mean vector

$$\underline{d}_{.....} = \begin{pmatrix} d_{.....1} \\ \vdots \\ d_{.....P} \end{pmatrix} = \frac{1}{IJKMN}\begin{pmatrix} \sum\limits_{ijkmn}\sum\sum\sum\sum d_{ijkmn1} \\ \vdots \\ \sum\limits_{ijkmn}\sum\sum\sum\sum d_{ijkmnP} \end{pmatrix}$$

and $\underline{\hat{V}}$ is the $(P \times P)$ sample covariance matrix

$$\underline{\hat{V}} = \left\{v_{pq}\right\} = \frac{1}{IJKMN}\left\{ \sum\limits_{ijkmn}\sum\sum\sum\sum d_{ijkmnp}d_{ijkmnq} - IJKMNd_{.....p}d_{.....q}\right\}$$

The hypothesis may be tested at an α significance level by comparing the observed T^2 value to the value

$$T_{\alpha}^2 = \frac{IJKMN-1}{IJKMN-P} F_{P,IJKMN-P,\alpha}$$

where $F_{P,IJKMN-P,\alpha}$ represents the upper α point of an F distribution with P and IJKMN-P degrees of freedom.

If $T^2 > T_\alpha^2$, the hypothesis that $E(\underline{d}_{ijkmn}) = \underline{0}$ is rejected, indicating that the proposed model is inadequate. In this situation, univariate tests on each d_{ijkmnp} and plots of these variables may reveal the source of the model inadequacy. If, on the other hand, $T^2 \leq T_\alpha^2$ this does not necessarily mean that the model is completely adequate, but only that there is not enough evidence in the data to disprove it. If a model does pass this test, the variability of the predicted values should be examined relative to the inherent variability in the observed dynamic response variables.